The Relationship Between Attitudes and Performance in Business Calculus

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Abstract

Undergraduate business curricula include coursework in both statistics and calculus, subjects that can be daunting to even capable students. This study examines undergraduate business students’ attitudes toward and performance in both business statistics and calculus, and determines that after controlling for ability, attitudes play a significant role in performance. Although self-reported attitudes become more positive over the course of the semester, attitudes toward calculus are less positive than those toward statistics, and negative attitudes are related to lower exam scores. For students with no prior calculus background, this relationship between negative attitudes and poor exam performance appears to be particularly strong. The conclusions have implications for increasing learning by improving attitudes.

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1. Introduction

In the mid 1990s, curriculum planners in the business school at a medium-sized U.S. comprehensive university recommended that a proof-based, one-semester calculus course taught in the math department no longer be required for business majors. Instead, instruction in basic calculus topics with more relevance to business applications would be provided by business faculty as an independent unit within an existing course.

Concurrent curriculum review led to a reorganization of the topics in the required business statistics, management science, and operations management classes. The result was the creation of three required one-semester (three credit hour) classes: a Business Statistics I class (covering descriptive statistics, probability, inference, and quality concepts), a Business Statistics II class (including regression analysis, forecasting, and decision analysis), and an operations management class (incorporating optimization in addition to typical OM topics). Staffing and curriculum considerations led to the decision to place calculus instruction within the second business statistics course; however, statistics instruction is not calculus-based.

The independent calculus unit within Business Statistics II occupies approximately four weeks, or twelve 50-minute class sessions, and comes at or near the end of the fourteen-week semester. Courses required later in the curriculum, such as finance and operations management, use basic calculus concepts and the Business Statistics II class is a prerequisite for them.

Multiple sections of both required statistics courses are offered each semester, are usually taught by tenured/tenure track faculty, and typically consist of 35-40 students each. Non-business majors, including math and economics majors, rarely enroll in the courses. Before enrolling in the first business statistics class, a student must have completed prerequisite courses in algebra and computer tools. All sections use the same textbook for both statistics classes, most recently Statistics for Business and Economics, 9e, by Anderson, Sweeney, and Williams.

Institutional research data (see First Year Experience Surveys) gathered from these business students indi-
cate that most are of traditional college age (over 90% entered college within one year of high school graduation) and many are first generation college students (fewer than 25% of them come from families where both parents have a college degree). Over 35% reported that they did well in high school without studying and another 12% reported not studying at all. When asked which academic area would be the hardest, 30-40% of them listed "math." This apprehension about math seems to confirm what instructors have observed and appears consistent with conclusions from the CRAFTY Curriculum Foundations Project of the MAA (2000):

Business students, although able, are often math phobic. Courses should strive to lessen math phobia, enable students to be more comfortable with mathematics, and help students appreciate the relevance of mathematics.

Consistent with this recommendation, the placement of calculus within the business curriculum rather than in the math department allows instructors to offer a more appropriate level of instruction (applications rather than proofs) and to provide more meaningful and relevant examples. Teaching materials focus on applications and the inexpensive and friendly text (Forgotten Calculus, 3e, by Bleau) includes a variety of examples and the solutions to all exercises. The calculus topics covered include functions, differentiation, unconstrained optimization of one variable, and simple integration. Yet despite the basic level of the material and the authors’ conscious efforts to provide active learning within a supportive environment, some students who have performed well on the course’s earlier multiple regression and business forecasting exams express apprehension about the study of calculus and score poorly on the calculus exam. This behavior has consistently frustrated instructors and has raised questions about the role of attitudes and math phobia in performance.

To this end, a study was devised to determine if the authors’ observations regarding the relationship between attitudes and performance were accurate. The unusual nature of the course provided a unique opportunity to differentiate, in an unchanging environment, attitudes toward and performance on exams in calculus as compared to other quantitative topics including business statistics. Before discussing the research questions, methods, and results of the study, a brief overview of theoretical background and relevant literature is given.

2. Theoretical Background

2.1. Ingredients of Learning

Before investigating how instructors can positively affect learning, one must understand the ingredients that contribute to learning, including those related to cognitive functions, environment, and behavior. As summarized by Kirton (2003), three cognitive functions are involved in an individual’s learning or problem solving capacity: cognitive effect, cognitive resource, and cognitive affect. Cognitive effect includes a person’s ability and preferred learning style. Cognitive resource refers to a person’s knowledge, skills and experience. Cognitive affect involves an individual’s attitudes, beliefs, values, needs and wants; it serves to direct a person’s motivation or energy to complete a task or achieve a goal. All three cognitive functions operate within an environment, called the social effect, that is external to the individual. Environment refers to culture, climate, and interaction with others. In academia, it includes the class setting, teaching techniques, and any feedback provided to the student (Kirton, 2003).

The actions that constitute an individual’s behavior are the product of many operating variables, including cognitive functions and the environment (Kirton, 2003). For example, cognitive functions, including ability, motivation, and attitudes, will determine whether someone is willing and able to perform a task. The environment has a reciprocal relationship with behavior. An individual’s behavior influences the environment, and behavior is adjusted as feedback from the environment is processed (Kirton, 2003).

Although ability is recognized as a major contributor to student learning, it is not the only important consideration. Many studies measure the effects of factors other than ability on learning in statistics and mathematics courses. Some focus on attitudes, while others include environmental influences. A brief overview of several studies follows.

2.2. Factors Associated with Achievement

Many studies have investigated the effects of factors such as attitudes, anxiety, and math background and ability on achievement in statistics courses (e.g. Roberts and Saxe, 1982; Harvey, Plake and Wise, 1985; Sutarso, 1992; Feinburg and Halperin, 1978; Adams and Hol-
Lalonde and Gardner (1993) investigated math background and ability, math and statistics anxiety, motivational intensity, and attitudes to see if they were related to achievement, and found that math aptitude lessens anxiety, which improves attitudes and motivation, which leads to greater effort, which leads to increased achievement. Tremblay, Gardner, and Heipel (2000) built upon Lalonde and Gardner’s model by adding motivational variables including interest in math and grades in previous courses. They found that low levels of aptitude, lack of interest in math, and unfavorable attitudes toward the course result in high levels of anxiety, which lead to poor performance.

2.3. Attitudes and Anxiety

Authors and instructors have observed and documented negative attitudes toward statistics, math and/or other quantitative courses, confirming the relationship between cognitive affect and learning. For example, Dillon (1982) reports that students felt "panicky," "sick," "worried," "frustrated," "doomed," and "overwhelmed" about having to take statistics. Some authors (e.g. Gal and Ginsburg, 1994; Gordon, 1995) suggest that these negative attitudes relate to high school math experiences, and that perceptions of statistics as not useful for jobs or future study may lead to feelings of dislike or disinterest in the subject. Widespread observations by instructors of statistics no doubt led to the creation of assessment instruments such as the SAS – Statistics Attitude Survey (Roberts and Bilderback, 1980) and the ATS – Attitudes Toward Statistics (Wise, 1985) that measure student competence, attitudes, interest, and beliefs about statistics.

Studies that have investigated statistics anxiety include Onwuegbuzie and Wilson (2003), who cite level of basic mathematical skill, extent of and success with prior mathematics courses, and mathematics anxiety as explanatory factors, and Schacht and Stewart (1992), who reference factors contributing to anxiety among students and suggest class activities to reduce it.

Some authors have suggested a link between statistics anxiety and math anxiety (e.g. Zimmer and Fuller, 1996). Tobias (1991) discusses steps in reducing math anxiety, concluding "My research with six hundred undergraduates proved, instead, that most average students have all the cognitive equipment they need to do advanced algebra, intermediate-level statistics, and college calculus. The problem is that they don’t believe they do." (p. 1)

2.4. The Effects of Environment on Learning

Many authors have investigated the role of environment on learning. Moos (1979) discusses how learning environments affect student satisfaction, learning, and personal growth. The author reports more success in a climate with high student involvement, personal student-teacher relationships, and innovative teaching methods.

Several authors have discussed or studied the effect of environment within college classes, and specifically in statistics courses. For example, Fraser and Fisher (1982) concluded that there were many significant correlations between outcomes and perceptions of environment. Gal and Ginsburg (1994) assert that learning statistics requires a supportive atmosphere where students feel safe, are motivated, and are comfortable with experimentation and inconclusive results. Tremblay, Gardner and Heipel (2000) concluded that attitude toward the instructor has a direct effect on motivation, which has positive effect on achievement. A similar finding was reported by Onwuegbuzie and Wilson (2003). Some authors suggest that the use of real-world situations, hands-on activities, or small group exercises can reduce anxiety, which should, in turn, increase satisfaction and achievement (e.g. Onwuegbuzie and Wilson, 2003; Mvududu, 2003; Schacht and Stewart, 1992; Garfield, 1993).

3. Purpose of This Study

Although there are multiple ingredients of learning, including ability (cognitive effect), preparation (cognitive resource), and environment (social effect), the focus of this study is on attitudinal factors (cognitive affect) that influence learning. In consideration of common sense and a large body of literature, it is conceded that ability, preparation, and environment have an effect on achievement. This study seeks to explore the existence and nature of the relationship between attitudes about calculus/quantitative courses and performance, after controlling for ability, preparation, and environment.

The instructors involved in this study strive to create a non-intimidating environment for the Business
Statistics II class. Students are encouraged to ask questions, are involved in hands-on activities, and are given real-world examples to motivate learning. Class sections of this course are small enough for instructors to know students' names and for students to become comfortable talking with their instructors both in and outside of the classroom. This atmosphere is intended to increase mutual trust and to encourage students to share their reactions about what they are learning. Based on conversations with students, the instructors' assessment of student engagement, and performance on exams, the authors have developed five informal observations about the relationship between attitudes and performance that this study seeks to validate:

• Observation 1: Student attitudes regarding calculus and quantitative classes become more positive over the course of the semester as students become more comfortable with the instructor and the curriculum; that is, at the onset students tend to anticipate that the material will be more difficult than it turns out to be.

• Observation 2: Students appear to have different attitudes toward calculus than they do about statistics and other quantitative classes. Some students who are performing well going into the calculus unit still express an inordinate amount of apprehension about calculus, feeling that it will be much more difficult than the statistics portion of the course.

• Observation 3: Even capable students may have negative attitudes that detract from their performance on exams. Frequent comments such as "I'm just not good at math" are sometimes a reflection of ability; however they appear to become a self-fulfilling prophecy when spoken by students who have appeared to follow along in class, to correctly complete assignments, and to thoroughly prepare for the exam.

• Observation 4: Expressions of anxiety and negative attitudes are more prevalent throughout the calculus unit than during the statistics portion of the course. Furthermore, these attitudes appear to coincide with lower average scores on the calculus exam compared to the statistics exams. This decrease in performance is widespread, including even capable students who were quite successful with the earlier statistics material. For example, almost 20% of students earning an A or B average on the statistics exams earn a D or an F on the calculus exam.

• Observation 5: Students who have never taken calculus before appear to have negative attitudes about the subject and are more likely to exhibit signs of "math anxiety" that impedes their performance on exams. They appear to anticipate that the study of calculus will be much more difficult than it is, even when the instructors try to allay their fears. To these students, the fear of the unknown appears to be great.

To determine the validity of these observations, several research questions were developed:

1. Do student attitudes improve significantly over the course of the semester?
2. Are student attitudes toward calculus distinct from their attitudes about statistics and other quantitative classes?
3. Are attitudes significant in predicting exam scores after controlling for ability, preparation, and environmental factors?
4. Is the relationship between attitudes and performance different and/or stronger for calculus than it is for statistics?
5. Is the relationship between attitudes and performance different for those who have not taken calculus than it is for those who have?

Methods used to investigate these questions are described in the next section.

4. Methods

This study, approved by the university's institutional review board, involved classes taught by the authors over three spring semesters. Data collected consisted of student records, performance on in-class exams, and a paper and pencil survey. Students present in class when the surveys were distributed overwhelmingly agreed to participate in the study, with only a handful of exceptions. After giving informed consent and agreeing to let the instructors access their student records and use their exam scores, these students filled out the survey during class time. Students who were absent from class or who subsequently dropped the course (approximately 14% and 2%, respectively, of
the students originally enrolled in the targeted classes), were not included in the study. Consistent with poor attendance, these students earned lower exam grades, on average, than those who did participate, and were more likely to be upperclassmen who were taking the class later than is customary.

Because of the unique placement of a calculus unit within a statistics course and the focus on calculus performance, it was decided that instruments specific to this course should be developed. The first cohort, consisting of 91 students enrolled in Spring 2002, completed three surveys over the course of the semester. One at the start of the term, one after the statistics portion of the class, and one at the end of the semester when the calculus unit was complete. The first survey asked background questions about the students' high school math experiences in addition to questions about current attitudes toward math, quantitative classes in general, and calculus. Subsequent surveys for the first cohort tracked only the attitudinal questions.

Additional cohorts were enrolled in the Spring semesters of 2003 and 2005. These students gave informed consent to access their student records and were administered only the first survey near the beginning of the semester. In each of these terms, 69 students participated, resulting in a total sample size of n = 229.

In the surveys, all attitudinal questions were measured on a 5-point Likert scale, with students marking one of five multiple-choice responses, labeled from (1) Disagree Strongly to (5) Agree Strongly. Students were asked to self-report usual math grades from high school (5 = As, 4 = Bs, 3 = Cs, 2 = Ds, 1 = Fs). In addition, student records were accessed to ascertain each student's SAT Math score (or ACT score converted to an SAT equivalent according to admissions practices) and grades in the prerequisite algebra class (measured on a standard 4-point scale). Each student's gender and class standing (sophomore, junior, or senior) were also recorded, and exam scores were provided by the instructor. Student scores on the calculus exam provided the measure of performance in calculus; performance in statistics was measured by the mean of the scores on the two midterm exams covering regression and forecasting. The variables included in the study are listed in Table 1.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SATMath</td>
<td>SAT math (or converted ACT) score from student records</td>
</tr>
<tr>
<td>MathGrade</td>
<td>Grade in prerequisite algebra class from student records (standard 4.0 scale)</td>
</tr>
<tr>
<td>HSGrades</td>
<td>Usual grades in high school math (self reported, 5 = As, 4 = Bs, 3 = Cs, 2 = Ds, 1 = Fs)</td>
</tr>
<tr>
<td>AttMathHS</td>
<td>My attitude toward math in high school was positive</td>
</tr>
<tr>
<td>EasyHS</td>
<td>Math was easy in high school</td>
</tr>
<tr>
<td>AttMathNow</td>
<td>My attitude toward current math/quantitative classes is positive</td>
</tr>
<tr>
<td>EasyNow</td>
<td>Math/quantitative classes are easy now</td>
</tr>
<tr>
<td>EasyCalc</td>
<td>The study of calculus will be easy</td>
</tr>
<tr>
<td>CalcComp</td>
<td>The study of calculus will be easy compared to other units in this class</td>
</tr>
<tr>
<td>CalcAtt</td>
<td>My attitude toward calculus is positive</td>
</tr>
<tr>
<td>CalcVal</td>
<td>Reaction to the statement &quot;Calculus is for mathematicians only and has no applications in business&quot; (from 1 = Agree Strongly to 5 = Disagree Strongly)</td>
</tr>
<tr>
<td>TaskCalc</td>
<td>Indicator variable (1 if have studied calculus before, 0 if not)</td>
</tr>
<tr>
<td>Instructor</td>
<td>Indicator variable for instructor</td>
</tr>
<tr>
<td>Term1, Term 3</td>
<td>Indicator variables indicating in which of three terms data collected</td>
</tr>
<tr>
<td>Female</td>
<td>Indicator variable (1 = female, 0 = male)</td>
</tr>
<tr>
<td>Upperclass</td>
<td>Indicator variable (1 = junior or senior, 0 = sophomore)</td>
</tr>
<tr>
<td>CalcExam</td>
<td>Score on calculus midterm (0 to 100)</td>
</tr>
<tr>
<td>StatsExams</td>
<td>Mean of scores on two statistics midterms (0 to 100)</td>
</tr>
</tbody>
</table>

Student records did not contain SAT Math and math grade data for some of the students. Of the 229 participants, 31 did not have SAT (nor converted ACT) information in their student records. An inspection of these student records indicated that these students were typically transfer, nontraditional, or international students. In addition, four students did not have a grade for the prerequisite math class. Three of these students were transfer students who were also missing
SAT scores. In all, 32 students (14%) were missing either SAT or a math grade or both.

An investigation of the differences between those missing and not missing data, which is discussed in detail in the Analysis and Results section, revealed only slight differences between the groups. It was desired to include the cases with missing values in the analysis so that the sample would more accurately reflect the population that includes such transfer, nontraditional, and international students. Using a method discussed in Cohen & Cohen (1975), missing values were replaced with the mean and a binary variable was created to indicate that these cases were missing data. This indicator variable was then included as an independent variable in analyses such as the regression models described below.

To explore research question 1, data from the first cohort that tracked changes in attitudes over the course of the semester were analyzed using paired t-tests and the Bowker test for symmetry (Bowker, 1948). To determine if attitudes toward calculus were distinct from attitudes toward other quantitative subjects (research question 2), a Principal Components Analysis was conducted on data from the first survey common to all three cohorts. These same data were used to develop multiple regression and Principal Components regression models to investigate the remaining questions regarding the effects of attitudes on performance. The results of the analyses are discussed below, and all are based on a 5% significance level unless otherwise specified.

5. Analysis and Results
5.1. Descriptive Statistics

Of the 229 students in the study, 51% were female and 38% had upperclass standing. The mean SAT Math score was 500, and 36% had earned an A in the prerequisite algebra class. Slightly more than 50% of the sample indicated that they liked their math classes in high school, and 28% reported taking a calculus course in high school. More than half of respondents (53%) indicated that they thought math was hard in high school. A summary of the descriptive statistics for the data set is shown in Table 2. In cases where relative frequencies are reported, "5" always corresponds to the most positive response.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StdDev</th>
<th>Min</th>
<th>Max</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>AttMathHS</td>
<td>229</td>
<td>3.39</td>
<td>1.34</td>
<td>1</td>
<td>5</td>
<td>10%</td>
<td>20%</td>
<td>17%</td>
<td>25%</td>
<td>27%</td>
</tr>
<tr>
<td>EasyHS</td>
<td>229</td>
<td>2.69</td>
<td>1.03</td>
<td>1</td>
<td>5</td>
<td>5%</td>
<td>45%</td>
<td>24%</td>
<td>19%</td>
<td>5%</td>
</tr>
<tr>
<td>AttMathNow</td>
<td>229</td>
<td>3.32</td>
<td>1.15</td>
<td>1</td>
<td>5</td>
<td>5%</td>
<td>21%</td>
<td>25%</td>
<td>31%</td>
<td>17%</td>
</tr>
<tr>
<td>EasyNow</td>
<td>229</td>
<td>2.58</td>
<td>1.00</td>
<td>1</td>
<td>5</td>
<td>10%</td>
<td>39%</td>
<td>22%</td>
<td>14%</td>
<td>5%</td>
</tr>
<tr>
<td>EasyCalc</td>
<td>229</td>
<td>2.55</td>
<td>0.89</td>
<td>1</td>
<td>5</td>
<td>9%</td>
<td>42%</td>
<td>38%</td>
<td>7%</td>
<td>3%</td>
</tr>
<tr>
<td>CalcComp</td>
<td>229</td>
<td>2.39</td>
<td>0.89</td>
<td>1</td>
<td>5</td>
<td>12%</td>
<td>49%</td>
<td>29%</td>
<td>7%</td>
<td>3%</td>
</tr>
<tr>
<td>CalcAtt</td>
<td>229</td>
<td>2.72</td>
<td>0.72</td>
<td>1</td>
<td>4</td>
<td>7%</td>
<td>24%</td>
<td>61%</td>
<td>9%</td>
<td>0%</td>
</tr>
<tr>
<td>CalcVal</td>
<td>229</td>
<td>3.48</td>
<td>1.02</td>
<td>1</td>
<td>5</td>
<td>4%</td>
<td>11%</td>
<td>24%</td>
<td>35%</td>
<td>15%</td>
</tr>
<tr>
<td>HSGrades</td>
<td>229</td>
<td>4.15</td>
<td>0.83</td>
<td>1</td>
<td>5</td>
<td>0%</td>
<td>2%</td>
<td>20%</td>
<td>30%</td>
<td>40%</td>
</tr>
<tr>
<td>SATMath</td>
<td>198</td>
<td>500.0</td>
<td>81.63</td>
<td>130</td>
<td>710</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MathGrade</td>
<td>225</td>
<td>3.13</td>
<td>0.92</td>
<td>1</td>
<td>4.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CalcExam</td>
<td>229</td>
<td>81.87</td>
<td>15.70</td>
<td>23</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>StatExams</td>
<td>229</td>
<td>83.47</td>
<td>13.43</td>
<td>30</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.2. Identification of Control Variables

When the differences by gender and upperclass (junior or senior) status were investigated using independent samples t-tests and chi-square tests for independence, women were found to have significantly higher math grades and exam scores than men, and exhibited more positive attitudes on several of the attitudinal survey questions (AttMathHS, AttMathNow, CalcAtt, CalcVal). These results are consistent with instructor observations that women have better attendance and study habits, and tend to be more conscientious about completing assignments than men.

Students with upperclass standing were found to have significantly lower math grades, exam scores, and SAT scores, and had indicated significantly more negative attitudes on survey questions relating to attitudes toward quantitative classes (AttMathHS, AttMathNow, EasyNow). Students who are on-track take the course in their sophomore year, so those who enroll later may...
have avoided the class, have had trouble meeting the prerequisites, or have been unsuccessful in a previous attempt. Thus, lower grades and more negative attitudes are not surprising.

Average exam scores of one instructor were significantly lower than exam scores of the other. A review of exams indicates that one instructor tended to create longer and more difficult exams. No other differences between instructors were found in any of the survey questions, math grades, or SAT scores.

When students enrolled in different terms are compared, ANOVAs show no significant differences in exam scores or SAT scores and chi-square tests for independence indicate no significant differences in grades, and only two significant differences in attitudes. Specifically, students enrolled in Spring 2002 felt that calculus would be significantly more difficult than students enrolled in Spring 2005 (as measured by the variables EasyCalc and CalcComp).

Chi-square tests and t-tests were used to compare students who had taken calculus to students who had not taken calculus. T-tests revealed that those who had taken calculus before had significantly higher exam scores and SAT scores, while chi-square tests showed they earned significantly higher grades in high school and in the prerequisite math course. Additional chi-square tests suggested that these students were more likely to believe that the calculus material would be easier; however, they did not feel that high school or current math classes were easier, nor did they have significantly different attitudes toward calculus than the students who had not taken it.

SAT scores and/or grades in the prerequisite algebra class were missing for some students. Chi-square and t-tests show no differences in attitudes, exam scores, and most survey questions between these students and others. The only difference identified was that those missing data were less likely to report earning As in high school (HSGrades).

Because of the differences found by gender, upperclass status, instructor, term, missing, and whether the students had taken calculus, these variables were included as control variables in the regressions used to investigate the research questions.

5.3. Research Question 1

To determine if student attitudes regarding quantitative classes and calculus improve significantly over the course of the semester, responses to survey questions were compared at various points during the semester. These data were gathered only for the 91 students participating in the Spring 2002 semester. Because a small number of these students did not take or fully complete either the second or third surveys, the sample sizes in the following analyses will differ.

When responses to current attitudes questions at the beginning of the semester are compared to those measured before the calculus unit using paired t-tests, only one significant difference was found: students have more positive attitudes about math/quantitative classes in the middle of the term (mean = 3.51) than they do at the beginning (mean = 3.21, T = -3.69, p < 0.001, n = 90, where a 5 indicates the most positive attitude).

On the other hand, several changes were found in attitudes when survey responses before the calculus unit are compared to those measured after the calculus unit (see Table 3, where 5 indicates the most positive attitude). In particular, significant positive changes were found in students’ attitude toward calculus (CalcAtt) and how easy they felt calculus to be (EasyCalc, CalcComp). To alleviate any concerns about the use of a test for means of ordinal data, even though the sample size is large, each of these significant differences was confirmed using a non-parametric test, the Bowker test of symmetry, which tests for non-random changes in subject responses to categorical data over time (see Bowker, 1948).

<table>
<thead>
<tr>
<th>CHANGES IN ATTITUDES FROM BEFORE TO AFTER CALCULUS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N Before Calculus</td>
</tr>
<tr>
<td>AntMathNow</td>
<td>85</td>
</tr>
<tr>
<td>EasyNow</td>
<td>85</td>
</tr>
<tr>
<td>CalcAtt</td>
<td>79</td>
</tr>
<tr>
<td>EasyCalc</td>
<td>84</td>
</tr>
<tr>
<td>CalcComp</td>
<td>83</td>
</tr>
<tr>
<td>CalcVal</td>
<td>82</td>
</tr>
</tbody>
</table>

5.4. Research Question 2

To determine if student attitudes toward calculus are distinct from their attitudes about other quantitative classes, a Principal Components Analysis was per-
formed on the independent variables in the study (excluding binary indicator variables). The analyses were first performed without 32 (of 229) cases with missing SATMath or MathGrade, and then again using all 229 cases with missing values replaced by the means. The results of the PCAs were nearly identical: the number of components extracted is the same, the percentage of variation explained is within 1%, and all of the individual items load most strongly on the same components with most factor loadings within .04 of their previous value. Therefore it was judged appropriate to replace the missing values by their means so that the data set would be as complete as possible.

Three components with eigenvalues over one were extracted, and a varimax rotation was applied. As Table 4 shows, there appear to be two components related to attitudes, one more strongly related to calculus attitudes, and one more strongly related to attitudes toward math/quantitative classes.

5.5. Research Question 3

To determine if attitudes are related to performance in calculus and statistics, regression analyses were used. First, two ordinary least squares (OLS) multiple regression models were run in which calculus exam score and the mean statistics exam score were the dependent variables. Independent variables included attitudinal and ability/preparation measures. Binary control variables for female, upperclass status, prior study of calculus, instructor, and the academic term were also included as independent variables. An additional binary variable was included to indicate whether missing data had been replaced by the mean.

As Table 5 indicates, some of the attitudinal variables were significant in predicting exam scores in both models. The variable representing high school math attitudes (AttMathHS) was significant in the statistics model, and the variable representing how easy math was in high school (EasyHS) was significant in the calculus model. Current attitudes toward quantitative classes (AttMathNow) was significant in both models. Also significant in predicting both types of exam scores were SAT Math scores, grades in the prerequisite math class (MathGrade) and in high school (HSGrades), while gender and instructor were only significant in the statistics model. The variable indicating whether the student had taken calculus before was not significant in either model, nor was the variable indicating a missing SAT score or math grade.

The models shown in Table 5 were examined for evidence of multicollinearity. The largest Variance Inflation Factor is 2.455 (tolerance = 0.407), which indicates some multicollinearity. Some of the coefficients appear to have opposite sign than one might expect. For example, in the calculus model, the negative coefficient of EasyHS indicates that the easier students found math in high school, the lower the expected calculus exam grade. In the statistics model, AttMathHS has a negative coefficient, indicating better attitudes in high school lead to lower expected exam grades. Additional evidence of multicollinearity results from the examination of the criteria put forth by Belsely, Kuh & Welsch (1980). A conditioning index in excess of 30 coupled with two variance proportions greater than 0.5 indicates that some multicollinarity is present.
In light of this finding, Principal Components Regression was used to test for the effects of the independent variables on calculus and statistics exam scores. Based on the Principal Components shown in Table 4, factor scores were created automatically by the statistical software using the regression approach (see e.g. Tabachniak & Fidell, 2001). For the purposes of the regression model, the components were named on the basis of the variables most strongly loading on each as follows:

- **Component 1: Calculus Attitudes**
- **Component 2: Quantitative Attitudes**
- **Component 3: Ability/Preparation**

Note that the final component, consisting of SAT Math score, grade in the prerequisite algebra class, and grades in high school, was named Ability/Preparation because it is difficult to separate the effects of ability and preparation within these measures. These three components were used for the attitudinal and ability/preparation variables in the Principal Components Regression model, along with the seven binary control variables. Missing data values were replaced with the means, giving a total of n = 229 records. The results of this analysis are shown in Table 6. The b_i and beta values represent, respectively, the unstandardized and standardized regression coefficients.

In both the calculus and statistics models, both the calculus attitudes and the quantitative attitudes components were significant. As expected, the ability/preparation component was significant, and showed a large effect on scores compared to the other variables. The coefficients of the (standardized) factors indicate that as the ability/preparation factor score increases by one standard deviation, then scores on the calculus exam could be expected to increase by 8.2 points out of 100, while statistics scores increase by an estimated 6.5 points. Increases of approximately 2 to 3 points can be expected in both models as the attitude factor scores increase by one standard deviation. Those who had taken calculus before did not score significantly higher than others when other variables are taken into account.

Also significant in both models were gender and instructor. The female variable had a significant positive effect in both models. An examination of gradebook data reveals that women enrolled in this course tend to have fewer absences, and are more consistent in completing and earning higher grades on assignments, which might shed light on the relationship between gender and exam scores.

The models also indicate that one instructor had significantly lower calculus and statistics exam scores. Possible reasons for this result include that this instructor's exams may also be more difficult or may be scored less generously. Another possibility is that students of this instructor are more likely to be upperclassmen (39% vs. only 21% of the other instructor's students) who might have delayed taking the course or are repeating it.
The Variance Inflation Factors for these models are all under 1.40 (tolerances above .71), so there is little evidence of multicollinearity. The regression assumptions appear to be met for these models except for some heteroscedasticity, which is likely to be caused by students with lower ability/preparation factor scores showing a wide range of exam scores, while students with high ability/preparation scores generally earn high exam scores with little variation.

To determine whether this heteroscedasticity has an effect on which variables are significant in the models, generalized least squares (GLS) regressions were run. In the calculus model, each of the variables found significant in the original model were also significant in the GLS model. With respect to the statistics model, the only substantive change in significance was that the quantitative attitudes component appeared only marginally significant (p = 0.081) in the GLS model; however, the significance of the calculus attitudes component remained.

Finally, to alleviate any concerns that replacing missing SAT Math scores with the mean has unduly affected the results, all analyses were rerun without the SAT Math variable. The Principal Components Analysis extracted slightly different components; however, the Principal Components regressions indicated that, although the R-squared values decreased substantially, a combined grades/attitudinal component and a perceived easiness component were both significant in predicting both calculus and statistics exam scores. This result also supports the result that attitudes are related to performance.

### 5.6. Research Question 4

To determine if the relationship between attitudes and performance is different and/or more pronounced for calculus than it is for statistics, the multiple and Principal Components regression models were examined. The differences in the OLS regressions (see Table 5) on the calculus and statistics scores were small: AttMathHS (attitude toward math in high school) was significant in the statistics model but not in the calculus model, whereas EasyMath (how easy math was in high school) was significant in the calculus model but not the statistics model. The Principal Components Regressions (see Table 6) indicated that both calculus attitudes and quantitative attitudes were significant in predicting calculus and statistics exam scores. These results still hold even when outliers (representing three students with extremely low calculus exam scores and two with extremely low statistics scores) were removed.

The results of Table 6 indicate that both the unstandardized and standardized regression coefficients for the attitudinal factors are larger in the calculus model than in the statistics model. Additional analyses reveal that both the zero-order correlations and the squared semi-partial correlations associated with the attitude factors are also larger in the calculus model (see Table 7). Although these differences are not large, they do indicate that the attitude factors have slightly more importance in the calculus model. Therefore, although relationship between attitudes and exam scores appears to be similar in nature for both calculus and statistics, attitudes do appear to play a slightly larger role in calculus performance.
5.7. Research Question 5

The final question to be investigated is whether the relationship between attitudes and performance is different for those who have and have not taken calculus before. In all of the regression models run, the indicator variable TookCalc was not significant, suggesting that when all other variables are controlled for, prior background in calculus does not have an effect on performance. To further investigate if attitudes have a different effect based on prior experience, the data set was split into two groups (TookCalc = 1 and TookCalc = 0) and the Principal Components Regressions were run separately for each group. The results for calculus scores are shown in Table 8.

For those students who had not taken calculus before, results indicate that in addition to ability and gender, both calculus and quantitative attitudes appear to have significant relationships with calculus exam scores. This model is almost identical to the overall model shown in Table 6, with the exception that the instructor variable is not significant here. One possible interpretation of this difference may be that students who have not taken calculus before find the exam to be difficult, regardless of the instructor.

For those students who had previous experience with calculus, neither attitudinal factor had a significant relationship with calculus exam scores; however, the ability factor was significant (see Table 8). These results may suggest that previous exposure to calculus can reduce anxiety and negative attitudes so that they do not detract from performance. As was found in the overall model, the instructor with more "upperclass" students appears to have lower scores on calculus exams among those students with previous calculus experience. Exam difficulty or scoring practices could be a factor in this result. The presence of a significant relationship between the "missing" variable and calculus exam scores might be explained by the fact that there were only 9 students who had taken calculus and had missing data. Upon closer examination, it was determined that the majority of these students were international students. It seems reasonable to infer that they may have had a different level of experience with...
calculus than domestic students had, which may contribute to their higher scores.

The fact that gender appeared significant in the "all student" model for calculus exam scores (refer to Table 6) and is not significant for students with previous calculus experience might be related to effort. As discussed previously, when all students are considered, there were significant differences between men and women with respect to the number of assignments missed, scores on homework assignments, and absences. However, when only students who had taken calculus before were compared by gender, no such differences were found.

When the statistics model for those who had taken calculus (not shown) and the overall statistics model (see Table 6) are compared, it was found that the ability factor and the gender and instructor variables were still significant, but that the two attitudinal factors were not. This result suggests that, for students with stronger quantitative backgrounds that include calculus, negative attitudes, if present, do not appear to be related to performance on statistics exams.

6. Discussion and Conclusions

The results of the statistical analyses indicated that overall, attitudes toward quantitative classes and calculus can improve over the course of the semester. There was a significant increase in positive attitudes about quantitative classes throughout the first part of the semester while the statistics modules are taught. This result may be explained, in part, by the supportive environment created by the instructors, who are accessible, provide real-world examples, and include hands-on activities. These improved attitudes may also be due to success early in the course, leading students to have increased confidence in their abilities.

After the calculus unit, a significant positive shift in the attitudes about calculus appeared (see Table 3). The most significant changes occurred for the EasyCalc and CalcComp variables, indicating that students anticipated that calculus would be more difficult than it actually was, supporting the idea of a calculus "phobia."

Results that do not support this finding include the fact that there was no significant change from the beginning to the end of the semester in how easy students felt quantitative classes were. It is interesting that, although students indicated liking quantitative classes significantly more before calculus than at the start of the term, there was a slight, marginally significant decrease is these attitudes after calculus (p-value = 0.067). This result appears to suggest that any progress in improving attitudes made in the beginning of the term while covering statistics might be partially negated by the calculus unit. Even though attitudes toward calculus improved, students expressed much less affinity for it than they did for quantitative classes in general.

The Principal Components Analysis supported the observation that students make a distinction between calculus and other quantitative classes. There appeared to be two components, one more strongly related to calculus attitudes, and one more strongly related to attitudes toward math/quantitative classes. Although some of the items loaded on both factors (e.g. AttMathNow and EasyNow), clearly not all of the items loaded on a single large attitudinal factor, indicating some difference in how the subjects are viewed by students.

When trying to determine the effect of attitudes on performance, it is clear that attitudinal variables are significant predictors, whether one relies on the OLS regressions or the Principal Components regressions. These results hold when three outliers identified in the calculus models and two in the statistics models were removed, with only slight changes in the p-values. They also hold when a GLS regression model was used to address heteroscedasticity, and when SAT scores were omitted from the analyses, although the attitudinal components took slightly different form. Thus, it is safe to assume that attitudes do have an impact on performance, although that effect is small compared to the effects of ability/preparation. This finding supports the instructors’ observations that even capable students may be hindered by apprehension.

The observation that attitudes have a larger effect on calculus performance than on statistics performance was supported, though not strongly, by these results. The differences in the OLS regressions were slight, and the same attitudinal components were significant in both the calculus and statistics Principal Components regressions. What can be said is that the attitudi-
nal factors have slightly more importance in the calculus model than in the statistics model, as measured by the standardized and unstandardized regression coefficients and the squared semi-partial and zero-order correlations.

Lastly, students who had not taken calculus before appeared to be more strongly affected by attitudes than students who had. For those without prior calculus experience, both the calculus and quantitative attitudes components were significant in the Principal Components regressions, in addition to the ability/preparation component. For those who had calculus before, only the ability/preparation factor appeared significant. Students who have already taken a calculus course are likely to be more talented and their performance appears to be unaffected by their attitudes, unlike their peers without prior calculus experience.

There are several limitations to this study. First of all, the study is limited by the availability of data to measure ability and preparation. Ideally, some sort of IQ score would be used to measure ability and a thorough understanding of each student’s high school and college math experiences would be represented in variables used for preparation. However, measures such as these are not readily available; instead, the SAT math score and grades were used to represent ability and preparation. These are admittedly imprecise and each is likely to reflect some combination of ability and preparation, as evidenced by the Principal Components Analysis. In addition, the use of grades introduces an unknown amount of variation into the study, as grades assigned by different instructors may be inconsistent.

Because the focus of this study is on performance, only students who completed the course are included and the results are not generalizable to all students who enroll. Future research may focus on the role of attitudes for those students who withdraw before completing the course.

While 36% to 44% of the variation in exam scores has been explained, there are other measures that could be expected to contribute to the prediction of performance on exams that were not included in this analysis. For example, measures of effort, motivation, or time spent preparing were not used. Future research should focus on the collection of these data, some of which might be obtained through student surveys, and how these variables contribute to the prediction of exam scores.

Despite these limitations, this study has been valuable to instructors by validating the relationship between attitudes and performance. Of the ingredients of learning, instructors have no influence over ability, very little over preparation (unless remedial help can be provided), but do have influence on attitudes and the environment. Within the environment created by these instructors, attitudes improved over the course of the semester, and improved attitudes are related to increased performance. Therefore, if instructors are aware of and address negative attitudes, especially among those with limited mathematical backgrounds, students may have more success with the course.

This research shows that negative attitudes and apprehension are negatively related to success, particularly for those with no prior calculus experience, lending credence to the CRAFTY observations about the presence of math anxiety in business students. Participants in this study were exposed to a brief, independent calculus module taught within a familiar environment; it seems likely that business students who take a more rigorous, full-semester calculus class with a new instructor would exhibit no less apprehension. Therefore, the conclusions of this study should be of interest to those who teach business calculus and statistics courses, no matter how those courses are structured.

References


First Year Experience surveys, http://irt2.indstate.edu/home/stats/surveys/surveys.htm (last accessed on June 7, 2005).


